Signaling

Principal is a firm that can't tell the productivity of different workers apart.

One might expect mechanisms to develop in the marketplace to help firms distinguish among workers.

Both the firms and the high-ability workers have incentives for this.

One such mechanism is signaling Spence (1973, 1974).

High-ability workers may have actions they can take to distinguish themselves from their low-ability counterparts.

Simplest example: workers can submit to some costless test that reliably reveals their type

In any subgame perfect Nash equilibrium all workers with ability greater than $\underline{\theta}$ will submit to the test and the market will aqhieve the full information outcome.

Any worker who chooses not to take the test will be correctly treated as being no better than the worst type of worker.

However, in many instances, no procedure exists that directly reveals a worker's type.

But the potential for signaling may still exist.

Two types of workers with productivities θ_H and θ_L, where $\theta_H > \theta_L > 0$ and $\lambda = \text{Prob}\ (\theta = \theta_H) \in (0,1)$.

Before entering the job market a worker can get some education, and the amount of education that a worker receives is observable.

Assume that education does nothing for a worker's productivity.

Cost of obtaining education level e for a type θ worker: $c(e, \theta)$

Assume $c_e(e, \theta) > 0, c_{ee}(e, \theta) > 0, c_\theta(e, \theta) < 0$ for all $e > 0$, and $c_{e\theta}(e, \theta) < 0$

Thus, both the cost and the marginal cost of education are assumed to be lower for high-ability workers

Cost may be of either monetary or psychic

For example, the work required to obtain a degree might be easier for a high-ability individual.

Let $u(w, e \mid (\theta) = w - c(e, \theta)$ denote the utility of a type θ worker who chooses education level e and receives wage w

Outside option: $r(\theta)$ by working at home.

This otherwise useless education may serve as a signal of unobservable worker productivity.

In particular, equilibria emerge in which high-productivity workers choose to get more education than low productivity workers

Firms correctly take differences in education levels as a signal of ability.

The welfare effects of signaling activities are generally ambiguous.

By revealing information about worker types, signaling can lead to a more efficient allocation of workers' labor, and in some instances to a Pareto improvement.

At the same time, because signaling activity is costly, workers' welfare may be reduced if they are compelled to engage in a high level of signaling activity to distinguish themselves.

For simplicity, assume $r(\theta_H) = r(\theta_L) = 0$

Then unique equilibrium that arises in the absence of the ability to signal has all workers employed by firms at a wage of $w^* = E[\theta]$ and is Pareto efficient.

Hence, our study of this case emphasizes the potential inefficiencies created by signaling.

With alternative assumptions about the function $r(\cdot)$, signaling may instead generate a Pareto improvement.

Initially, a random move of nature determines whether a worker is of high or low ability.

Then, conditional on her type, the worker chooses how much education to obtain.

After obtaining her chosen education level, the worker enters the job market.

Conditional on the observed education level of the worker, two firms simultaneously make wage offers to her.

Finally, the worker decides whether to work for a firm and, if so, which one.

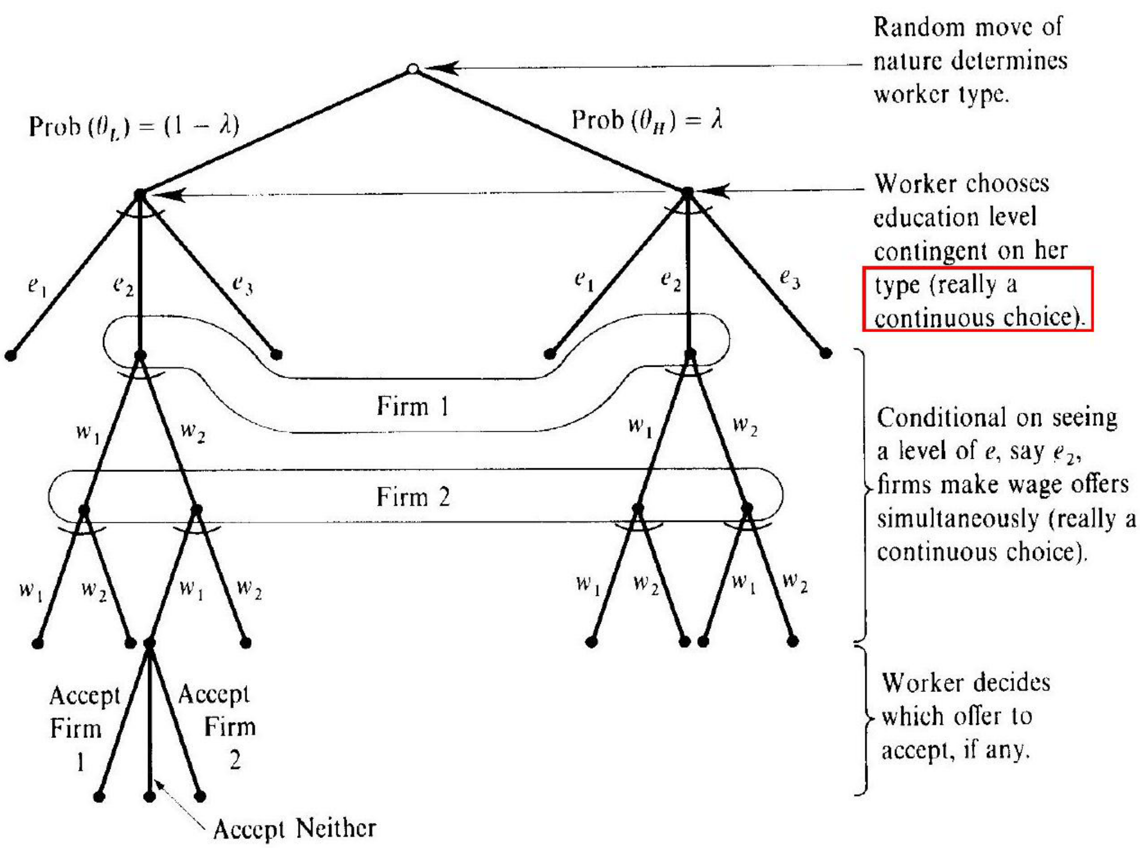


Figure 13.C. 1: The extensive form of the education signaling game.

Only a single worker of unknown type

> Many workers can be thought of as simply having many of these single-worker games going on simultaneously, with the fraction of high-ability workers in the market being λ.

Equilibrium concept: weak perfect Bayesian equilibrium, but with an added condition.

> Firms' beliefs have the property that, for each possible choice of e, there exists a number $\mu(e) \in [0,1]$ such that:
>
> (i) firm 1's belief that the worker is of type θ_H after seeing her choose e is $\mu(e)$
>
> (ii) after the worker has chosen e, firm 2's belief that the worker is of type θ_H and that firm 1 has chosen wage offer w is precisely $\mu(e)\sigma_1^*(w \mid e)$, where $\sigma_1^*(w \mid e)$ is firm 1's equilibrium probability of choosing wage offer w after observing education level e.
>
> Commonality to the firms' beliefs about the type of worker who has chosen e
>
> Firms' beliefs about each others' wage offers following e are consistent with the equilibrium strategies both on and off the equilibrium path.

We refer to a weak perfect Bayesian equilibrium satisfying this extra condition on beliefs as a perfect Bayesian equilibrium (PBE).

This PBE notion can more easily, and equivalently, be stated as follows:

A set of strategies and a belief function $\mu(e) \in [0,1]$ giving the firms' common probability assessment that the worker is of high ability after observing education level e is a PBE if

(i) The worker's strategy is optimal given the firm's strategies.

(ii) The belief function $\mu(e)$ is derived from the worker's strategy using Bayes' rule where possible.

(iii) The firms' wage offers following each choice e constitute a Nash equilibrium of the simultaneous-move wage offer game in which the probability that the worker is of high ability is $\mu(e)$

In the context of the model studied here, this notion of a PBE is equivalent to the sequential equilibrium concept discussed in Section 9.C.

We also restrict our attention throughout to pure strategy equilibria.

We begin our analysis at the end of the game.

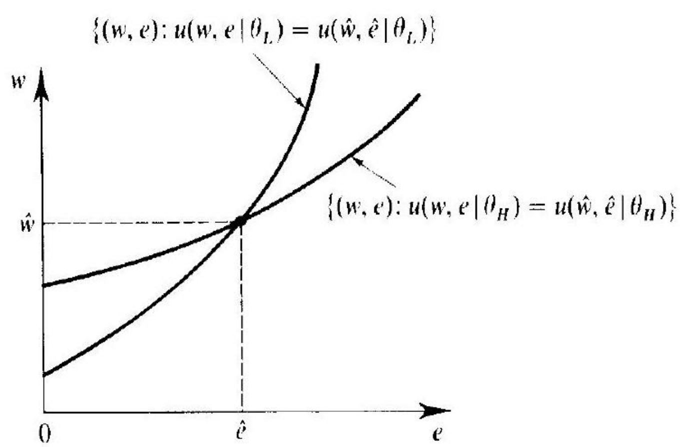
Suppose that after seeing some education level e, the firms attach a probability of $\mu(e)$ that the worker is type θ_H.

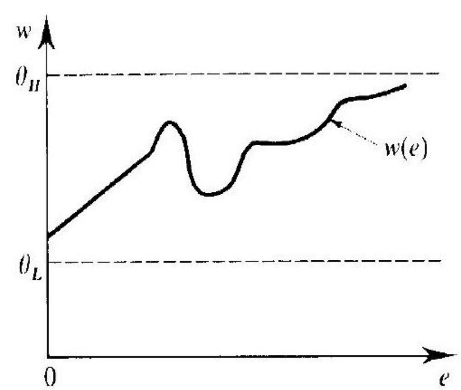
If so, the expected productivity of the worker is $\mu(e)\theta_H + (1 - \mu(e))\theta_L$.

In a simultaneous-move wage offer game, the firms' (pure strategy) Nash equilibrium wage offers equal the worker's expected productivity.

Thus, in any (pure strategy) PBE, we must have both firms offering a wage exactly equal to the worker's expected productivity:

$$w = \mu(e)\theta_H + (1 - \mu(e))\theta_L$$





Turn to the issue of the worker's equilibrium strategy, her choice of an education level contingent on her type.

Examine the worker's preferences over (wage rate, education level) pairs.

Figure 13.C. 2 depicts an indifference curve for each of the two types of workers (with wages measured on the vertical axis and education levels measured on the horizontal axis).

> Indifference curves cross only once and that, where they do, the indifference curve of the high-ability worker has a smaller slope.
>
> This is the single-crossing property, and plays an important role in the analysis of signaling models and in models of asymmetric information more generally.
>
> It arises here because the worker's marginal rate of substitution between wages and education at any given (w, e) pair is $(dw/de)_u = c_e(e, \theta)$, which is decreasing in θ because $c_{c,0}(e, 0) < 0$.

We can also graph a function giving the equilibrium wage offer that results for each education level, which we denote by $w(e)$.

Since in any PBE $w(e) = \mu(e)\theta_L + (1 - \mu(e))\theta_L$ for the equilibrium belief function $\mu(e)$, the equilibrium wage offer resulting from any choice of e must lie in the interval $[\theta_L, \theta_H]$.

> A possible wage offer function $w(e)$ is shown in Figure 13.C.3.

It is useful to consider separately two different types of equilibria that might arise:

> Separating equilibria, in which the two types of workers choose different education levels
>
> Pooling equilibria, in which the two types choose the same education level.

Separating Equilibria

Let $e^*(\theta)$ be the worker's equilibrium education choice as a function of her type, and let $w^*(e)$ be the firms' equilibrium wage offer as a function of the worker's education level.

Lemma 13.C.1: In any separating perfect Bayesian equilibrium, $w^*\left(e^*(\theta_H)\right) = \theta_H$ and $w^*\left(e^*(\theta_L)\right) = \theta_L$; that is, each worker type receives a wage equal to her productivity level

Proof:

In any PBE, beliefs on the equilibrium path must be correctly derived from the equilibrium strategies using Bayes' rule.

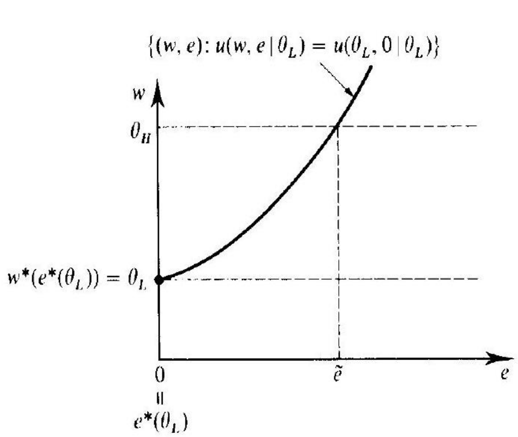
Here this implies that upon seeing education level $e^*(\theta_L)$, firms must assign probability one to the worker being type θ_L.

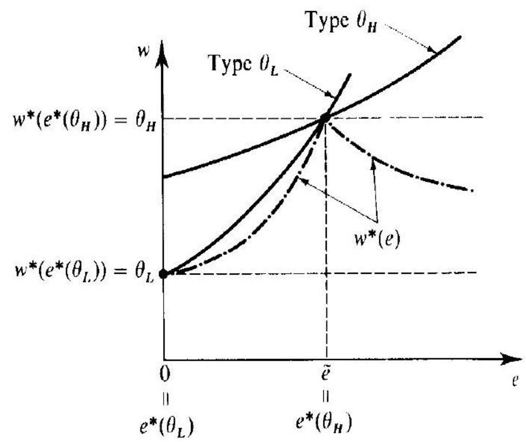
Likewise, upon seeing education level $e^*(\theta_H)$, firms must assign probability one to the worker being type θ_H. The resulting wages are then exactly θ_L and θ_H, respectively.

QED.

Figure 13.C. 2 (left): Indifference curves for high- and low-ability workers: the single-crossing property.

Figure 13.C. 3 (right): A wage schedule.





Lemma 13.C.2: In any separating perfect Bayesian equilibrium, $e^*(\theta_L) = 0$; that is, a low-ability worker chooses to get no education.

Proof:

Suppose not, that is, that when the worker is type θ_L, she chooses some strictly positive education level $\hat{e} > 0$.

According to Lemma 13.C.1, by doing so, the worker receives a wage equal to θ_L.

However, she would receive a wage of at least θ_L if she instead chose $e = 0$.

Since choosing $e = 0$ would have save her the cost of education, she would be strictly better off by doing so, which is a contradiction to the assumption that $\hat{e} > 0$ is her equilibrium education level.

QED.

Lemma 13.C. 2 implies that, in any separating equilibrium, type θ_L 's indifference curve through her equilibrium level of education and wage must look as depicted in Figure 13.C.4.

Using Figure 13.C.4, we can construct a separating equilibrium as follows:

$$e^*(\theta_H) = \tilde{e}$$

$$e^*(\theta_L) = 0$$

Let $w^*(e)$ be as drawn in Figure 13.C.5

The firms' equilibrium beliefs following education choice e are:

$$\mu^*(e) = (w^*(e) - \theta_L)/(\theta_H - \theta_L)$$

Note that they satisfy $\mu^*(e) \in [0,1]$ for all $e \geq 0$, since $w^*(e) \in [\theta_L, \theta_H]$.

To verify that this is indeed a PBE, note that we are completely free to let firms have any beliefs when e is neither 0 nor $\tilde{e}$.

On the other hand, we must have $\mu(0) = 0$ and $\mu(\tilde{e}) = 1$.

The wage offers drawn, which have $w^*(0) = \theta_L$ and $w^*(\tilde{e}) = \theta_H$, reflect exactly these beliefs.

What about the worker's strategy?

> Given the wage function $w^*(e)$, the worker is maximizing her utility by choosing $e = 0$ when she is type θ_L and by choosing $e = \tilde{e}$ when she is type θ_H.
>
> For each type that she may be, the worker's indifference curve is at its highest-possible level along the schedule $w^*(e)$.

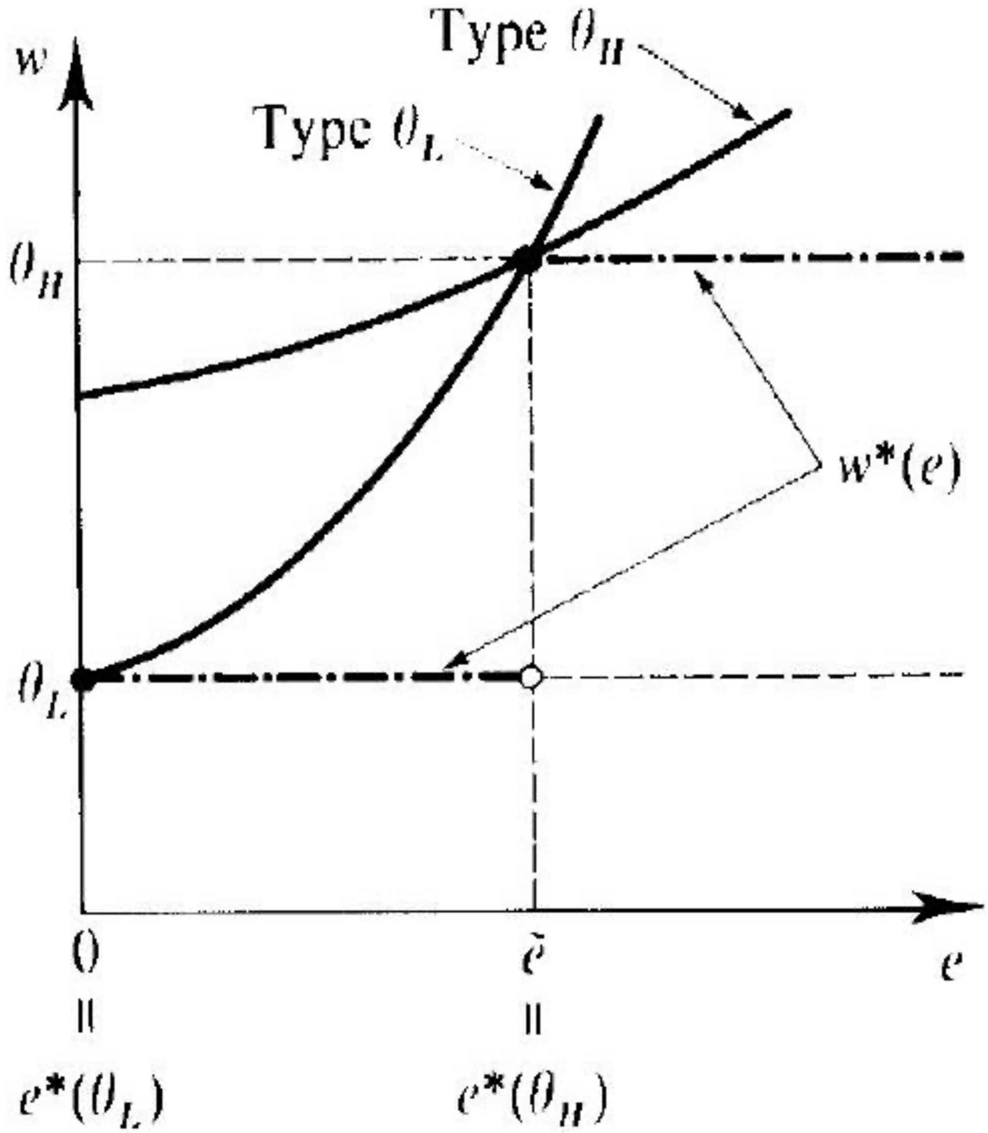
Thus, strategies $[e^*(\theta), w^*(e)]$ and the associated beliefs $\mu(e)$ of the firms do in fact constitute a PBE.

This is NOT the only PBE involving these education choices by the two types of workers.

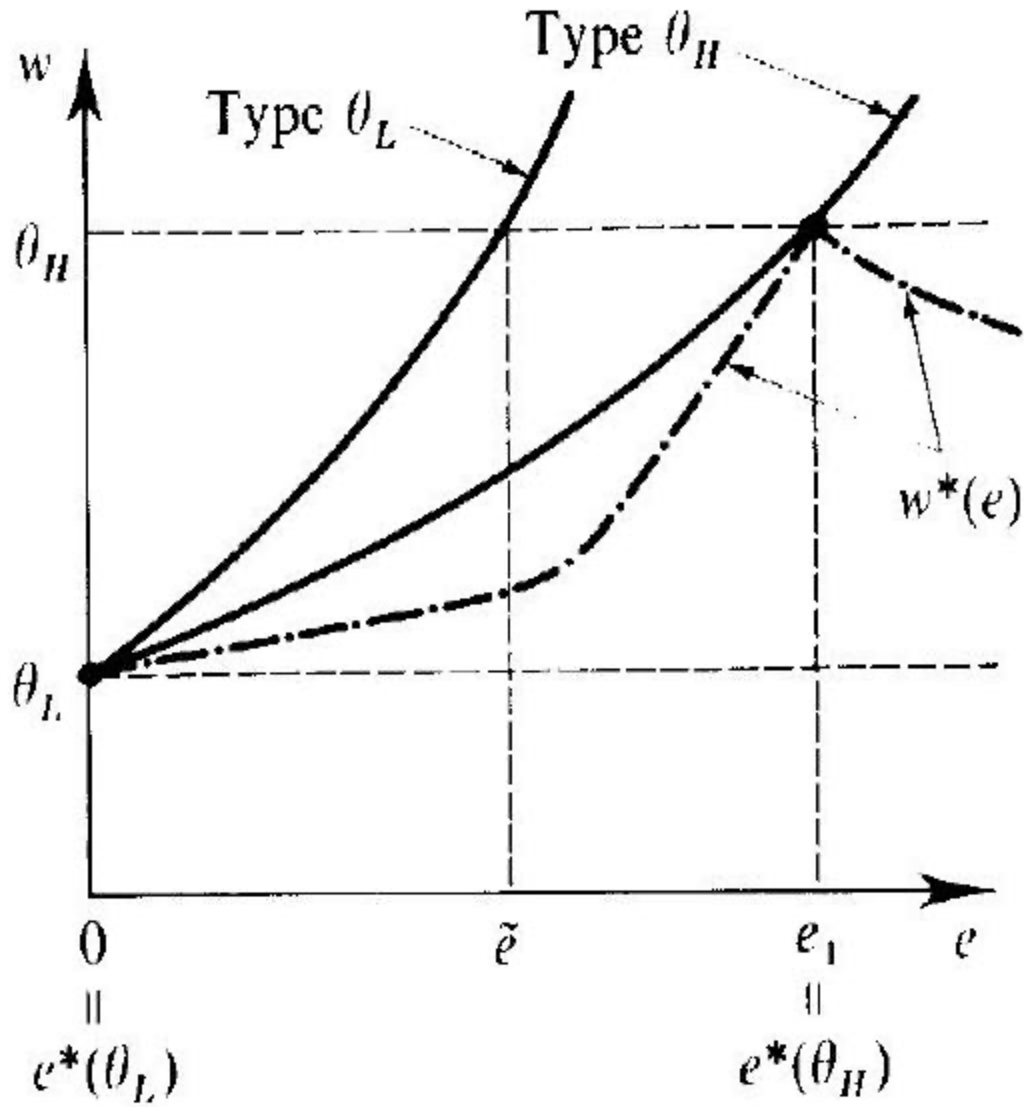
Because we have so much freedom to choose the firms' beliefs off the equilibrium path, many wage schedules can arise that support these education

Figure 13.C. 4 (left): Low-ability worker's outcome in a separating equilibrium.

Figure 13.C. 5 (right): A separating equilibrium: Type is inferred from education level.



Type θ_H
Type θ_L
w
θ_H
θ_L
$w^*(e)$
0
$\tilde{e}$
e
$\|$
$\|$
$e^*(\theta_L)$
$e^*(\theta_H)$



Flgure 13.C. 6 (left): A separating equilibrium with the same education choices as in Figure 13.C. 5 but different off-equilibriumpath beliefs.

Figure 13.C.7 (right): A separating equilibrium with an education choice $e^*(\theta_{\mathrm{H}}) > \tilde{e}$ by high-ability workers. choices.

Figure 13.C.6 depicts another one; in this PBE, firms believe that the worker is certain to be of high quality if $e \geq \tilde{e}$ and is certain to be of low quality if $e < \tilde{e}$

The resulting wage schedule has $w^*(e) = \theta_\mu$ if $e \geq \tilde{e}$ and $w^*(e) = \theta_L$ if $e < \tilde{e}$.

In these separating equilibria, high-ability workers are willing to get otherwise useless education simply because it allows them to distinguish themselves from low-ability workers and receive higher wages.

The fundamental reason that education can serve as a signal here is that the marginal cost of education depends on a worker's type.

Because the marginal cost of education is higher for a low-ability worker [since $c_{e\theta}(e, \theta) < 0$]...

a type θ_H worker may find it worthwhile to get some positive level of education $e' > 0$ to raise her wage by some amount $\Delta w > 0$...

...whereas a type θ_L worker may be unwilling to get this same level of education in return for the same wage increase.

As a result, firms can reasonably come to regard education level as a signal of worker quality.

The education level for the high-ability type observed above is not the only one that can arise in a separating equilibrium in this model.

Many education levels for the high-ability type are possible.

Any education level between $\tilde{e}$ and e_1 in Figure 13.C.7 can be the equilibrium education level of the high-ability workers.

A wage schedule that supports education level $e^*(\theta_H) = e_1$ is depicted in the figure.

Note that the education level of the high-ability worker cannot be below $\tilde{e}$ in a separating equilibrium because, if it were, the low-ability worker would deviate and pretend to be of high ability by choosing the high-ability education level.

On the other hand, the education level of the high-ability worker cannot be above e_1 because, if it were, the high-ability worker would prefer to get no education, even if this resulted in her being thought to be of low ability.

These various separating equilibria can be Pareto ranked.

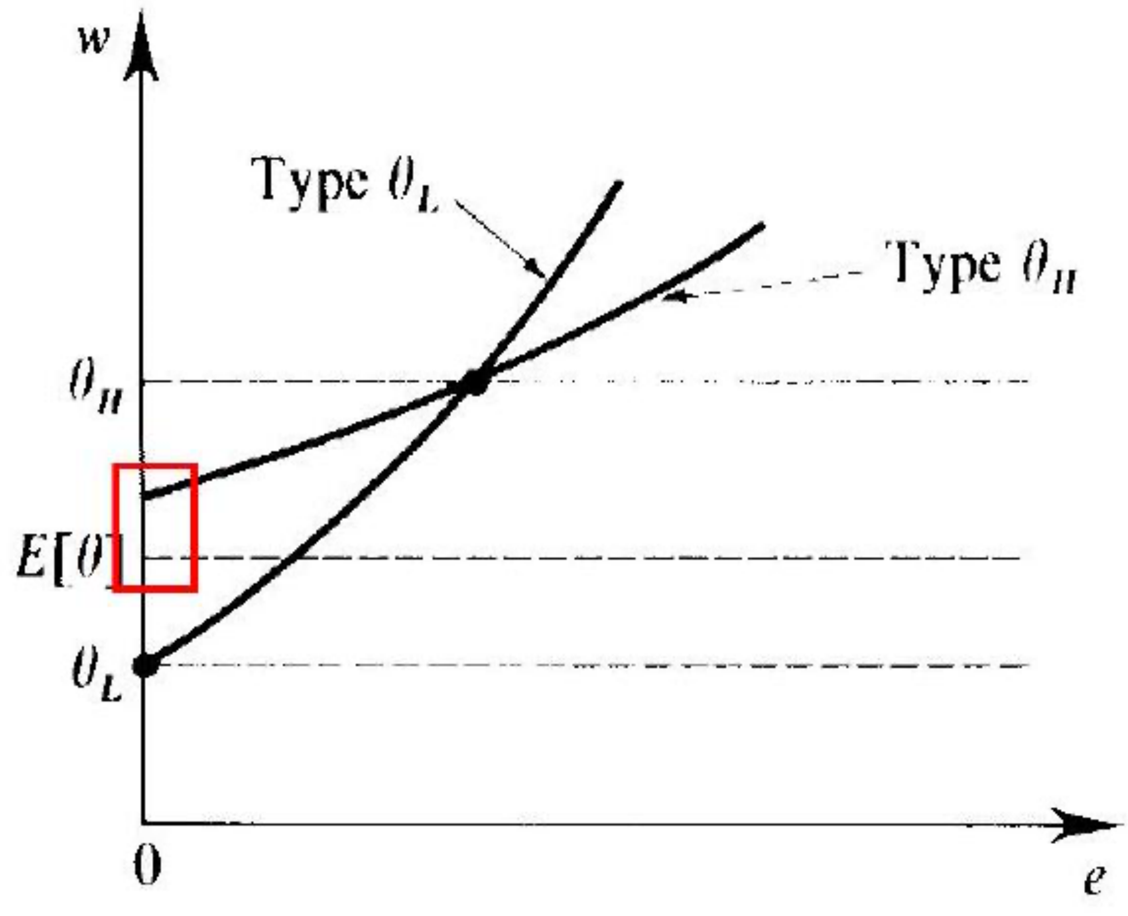
In all of them, firms earn zero profits, and a low-ability worker's utility is θ_L.

However, a high-ability worker does strictly better in equilibria in which she gets a lower level of education.

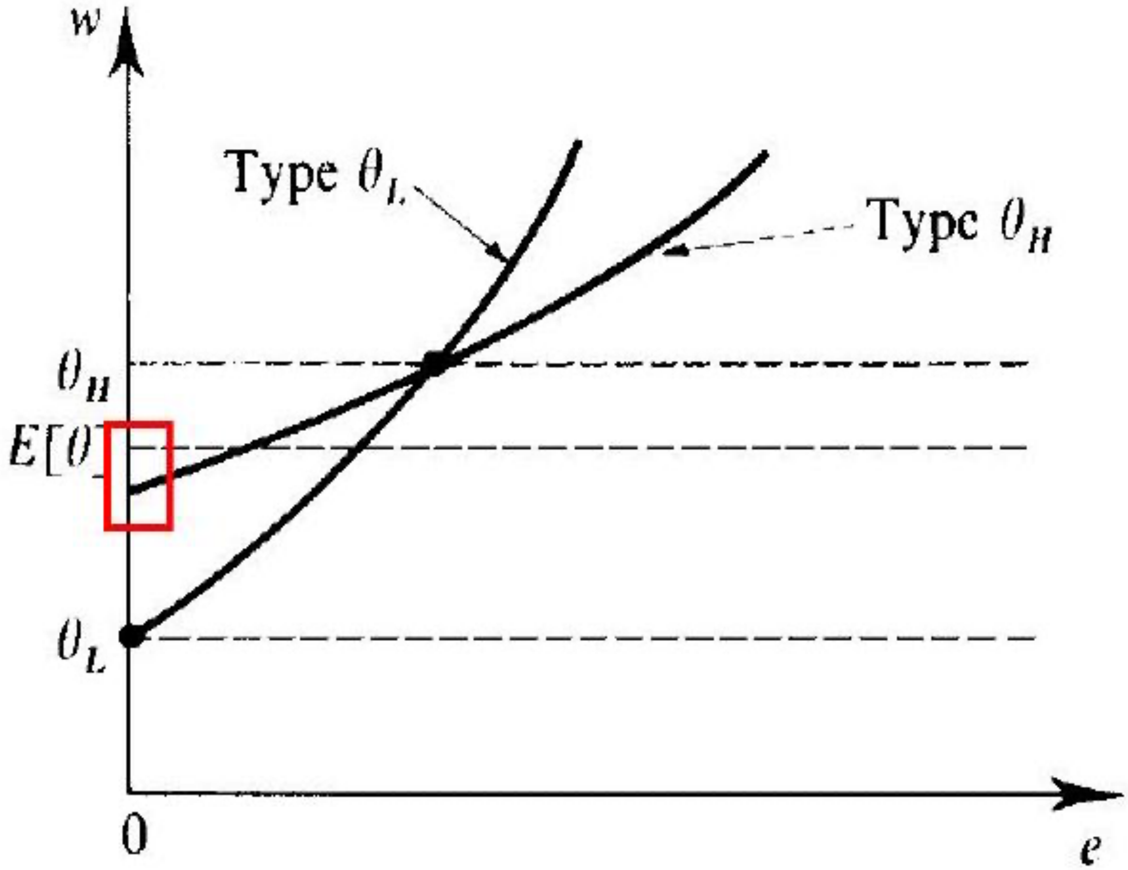
Thus, separating equilibria in which the high-ability worker gets education level $\tilde{e}$ (e.g., the equilibria depicted in Figures 13.C. 5 and 13.C.6) Pareto dominate all the others.

The Pareto-dominated equilibria are sustained because of the high-ability worker's fear that if she chooses a lower level of education than that prescribed in the equilibrium firms will believe that she is not a high-ability worker.

These beliefs can be maintained because in equilibrium they are never disconfirmed.



(a)



(b)
Figure 13.C. 8

Separating equilibria may be Pareto dominated by the no-signaling outcome. (a) A separating equilibrium that is not Pareto dominated by the no-signaling outcome.

(b) A separating equilibrium that is Pareto dominated by the no-signaling outcome.

When education is not available as a signal (so workers also incur no education costs)...

> Firms earn expected profits of zero
>
> However, low-ability workers are strictly worse off when signaling is possible.
>
> They incur no education costs, but when signaling is possible they receive a wage of θ, rather than $E(\theta)$.

What about high-ability workers?

The somewhat surprising answer is that high-ability workers may be either better or worse off when signaling is possible.

In Figure 13.C.8(a), the high-ability workers are better off because of the increase in their wages arising through signaling.

However, in Figure 13.C.8(b), even though high-ability workers seek to take advantage of the signaling mechanism to distinguish themselves, they are worse off than when signaling is impossible!

Although this may seem paradoxical (if high-ability workers choose to signal, how can they be worse off?), its cause lies in the fact that in a separating signaling equilibrium firms" expectations are such that the wage education outcome from the no-signaling situation, $(w, e) = (E[0], 0)$, is no longer available to the high-ability workers

> If they get no education in the separating signaling equilibrium, they are thought to be of low ability and offered a wage of θ_L.
>
> Thus, they can be worse off when signaling is possible, even though they are choosing to signal.

Note that because the set of separating equilibria is completely unaffected by the fraction λ of high-ability workers, as this fraction grows it becomes more likely that the high-ability workers are made worse off by the possibility of signaling

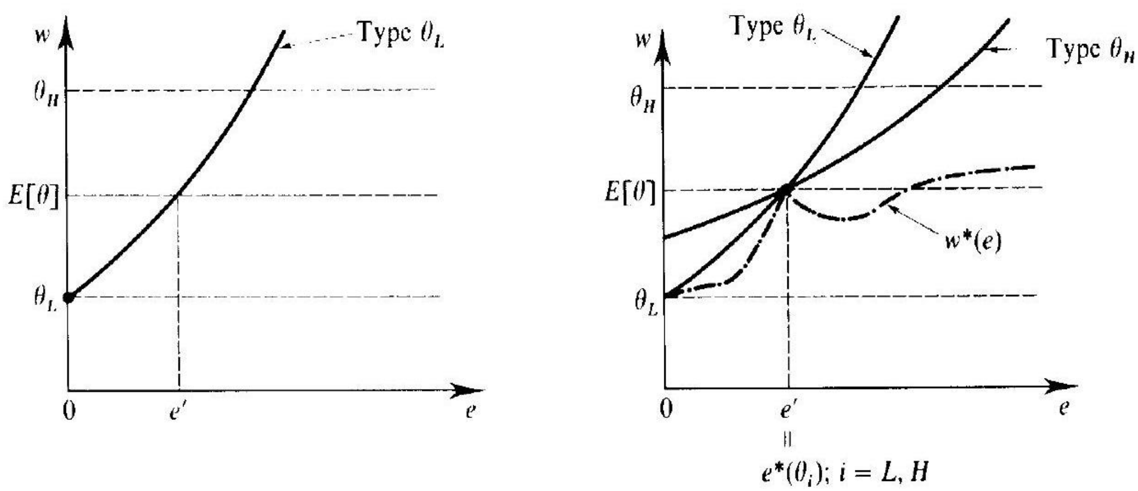
> As this fraction gets close to 1 , nearly every worker is getting costly education just to avoid being thought to be one of the handful of bad workers!

Pooling Equilibria

Consider now pooling equilibria, in which the two types of workers choose the same level of education, $e^*(\theta_L) = e^*(\theta_I) = e^*$.

Since the firms' beliefs must be correctly derived from the equilibrium strategies and Bayes' rulc when possible, their beliefs when they see education level e^* must assign probability λ to the worker being type θ_H.

Thus, in any pooling equilibrium, we must have $w^*(e^*) = \lambda\theta_H + (1 - \lambda)\theta_L = E[\theta]$.



The only remaining issue therefore concerns what levels of education can arise in a pooling equilibrium.

It turns out that any education level between 0 and the level e' depicted in Figure 13.C. 9 can be sustained.

Figure 13.C. 10 shows an equilibrium supporting education level e'.

Given the wage schedule depicted, each type of worker maximizes her payoff by choosing education level e'.

This wage schedule is consistent with Bayesian updating on the equilibrium path because it gives a wage offer of $E[\theta]$ when education level e' is observed.

Education levels between 0 and e' can be supported in a similar manner.

> Education levels greater than e' cannot be sustained because a low-ability worker would rather set $e = 0$ than $e > e'$ even if this results in a wage payment of θ_L.

A pooling equilibrium in which both types of worker get no education Pareto dominates any pooling equilibrium with a positive education level.

Once again, the Pareto-dominated pooling equilibria are sustained by the worker's fear that a deviation will lcad firms to have an unfavorable impression of her ability.

A pooling equilibrium in which both types of worker obtain no education results in exactly the same outcome as that which arises in the absence of an ability to signal.

> Thus, pooling equilibria are (weakly) Pareto dominated by the no-signaling outcome.

Multiple Equilibria and Equilibrium Refinement

We can have separating equilibria in which firms learn the worker's type, but we can also have pooling equilibria where they do not

> Within each type of equilibrium, many different equilibrium levels of education can arise.

In large part, this multiplicity stems from the great freedom that we have to choose beliefs off the equilibrium path.

To see a simple example of this kind of reasoning, consider the separating equilibrium depicted in Figure 13.C.7.

To sustain e_1 as the equilibrium education level of high-ability workers, firms must believe that any worker with an education level below e_1 has a positive probability of being of type θ_L.

But consider any education level $\hat{e} \in (\tilde{e}, e_1)$.

A type θ_L worker could never be made better off choosing such an education level than she is getting education level $e = 0$ regardless of what firms believe about her as a result.

Hence, any belief by firms upon seeing education level $\hat{e} > \tilde{e}$ other than $\mu(\hat{e}) = 1$ seems unreasonable.

But if this is so, then we must have $w(\hat{e}) = \theta_H$

So the high-ability worker would deviate to $\hat{e}$.

The only education level that can be chosen by type θ_H workers in a separating equilibrium involving reasonable beliefs is $\tilde{e}$.

Figure 13.C. 9 (left): The highest-possible education level in a pooling equilibrium.

Figure 13.C. 10 (right) A pooling equilibrium.

REASONABLE-BELIEFS REFINEMENTS IN SIGNALING GAMES

Consider the following class of signaling games: There are I players plus nature. The first move of the game is nature's, who picks a "type" for player 1 , $\theta \in \Theta = \{\theta_1, \ldots, \theta_N\}$. The probability of type θ is $f(\theta)$, and this is common knowledge among the players. However, only player 1 observes θ. The second move is player 1 's, who picks an action a from set A after observing θ. Then, after seeing player 1's action choice (but not her type), each player $i = 2, \ldots, I$ simultaneously chooses an action s_i from set S_i. We define $S = S_2 \times \cdots \times S_I$. If player 1 is of type θ, her utility from choosing action a and having players $2, \ldots, I$ choose $s = (s_2, \ldots, s_I)$ is $u_1(a, s, (0)$. Player $i \neq 1$ receives payoff $u_i(a, s, \theta)$ in this event. A perfect Bayesian
equilibrium (PBE) in the sense used in Section 13.C is a profile of strategies $(a(\theta), s_2(a), \ldots, s_1(a))$, combined with a common belief function $\mu(\theta \mid a)$ for players $2, \ldots, I$ that assigns a probability $\mu(\theta \mid a)$ to type θ of player 1 conditional on observing action $a \in A$, such that

(i) Player 1's strategy is optimal given the strategies of players $2, \ldots, I$.

(ii) The belief function $\mu(\theta \mid a)$ is derived from player 1's strategy using Bayes' rule where possible.

(iii) The strategies of players $2, \ldots, I$ specify actions following each choice $a \in A$ that constitute a Nash equilibrium of the simultaneous-move game in which the probability that player 1 is of type θ is $\mu(\theta \mid a)$ for all $\theta \in \Theta$.

In the context of the model under study here, this notion of a PBE is equivalent to the sequential equilibrium notion.